

## Magneto hydrodynamic couple Stress Peristaltic flow of blood Through Porous medium in a Flexible Channel at low Reynolds Number

<sup>a</sup>S.Ravi Kumar, <sup>a</sup>Ashfar Ahmed

<sup>a</sup>Assistant Professor, Department of Mathematics, NBKR Institute of Science and Technology (An Autonomous Institution, Accredited by NBA and 'A'- Grade of NAAC, ISO 9001:2008 Certified), Vidyanagar, SPSR Nellore, Andhra Pradesh, India.Pin-524413.

### Abstract

The present paper investigates the Magneto hydrodynamic couple stress peristaltic flow of blood through porous medium in a flexible channel at low Reynolds number. A perturbation method of solution is obtained in terms of wall slope parameter and closed form of expressions has been derived for axial velocity and transverse velocity and also sheare stress. The effects of various physical parameters on axial velocity, transverse velocity have been computed numerically. The resulting velocity in the converging (constricted) part of the channel is directed towards the boundary with the fluid moving in clockwise direction while in the dilated part, it moves in the anti-clockwise sense with resulting velocity directed towards the axis of the channel.

**KEYWORDS:** Peristaltic fluid flow, Couple stress fluids, Reynolds number, Magneto hydrodynamic, Porous medium.

### 1. Introduction

Pumping of fluids through flexible tubes by means of the peristaltic wave motion of the tube wall has been the subject of engineering and scientific research for over four decades. Engineers and physiologists term the phenomenon of such flow as peristalsis. It is a form of fluid transport induced by a progressive wave of area contraction or expansion along the walls of a distensible duct containing a liquid or mixture. Besides its various engineering applications (e.g., heart-lung machines, finger and roller pumps, etc.), it is known to be significant mechanism responsible for fluid transport in many biological organs including in swallowing food through esophagus, urine transport from kidney to bladder through thureter, movement of chyme in gastrointestinal tract, transport of spermatozoa in the ductus efferentes of the male reproductive tracts, and, in cervical canal, in movement of ovum in the female fallopian tubes, transport of lymph in lymphatic vessels, and in the vasomotion of small blood vessels such as arterioles, venules and capillaries.

M.S. Thesis. Shapiro et al. (1969) and Jaffrin and Shapiro (1971) explained the basic principles and brought out clearly the significance of the various parameters governing the flow. The literature on the topic is quite extensive by now and a review of much of the literature up to the year 1983, arranged according to the geometry, the fluid, the Reynolds number, the wave number, the amplitude ratio and the wave shape was presented in an excellent article by Srivastava and Srivastava (1984).

The study of peristalsis has received considerable attention in last three decades mainly because of its importance in biological systems and industrial applications. Several investigators have analyzed the peristaltic motion of both Newtonian and non-Newtonian fluids in mechanical as well as physiological systems (Fung and Yih (1968), Burns and Parkes (1969), Shapiro et al. (1969), Selverov and Stone (2001), Xiao and Damodaran(2001)

,Misra and Rao (2003), Radhakrishnamacharya and Srinivasulu (2007), Maruthi Prasad and Radhakrishnamacharya (2007), Muthu et al. (2007)). Peristaltic transport in non-uniform ducts is considerable interest as many channels in engineering and physiological problems are known to be of non-uniform cross-section. Srivastava et al. (1983) and Srivastava and Srivastava (1988) studied peristaltic transport of Newtonian and non-Newtonian fluids in non-uniform geometries. Radhakrishnamacharya and Radhakrishna Murthy (1993) studied the interaction between peristalsis and heat transfer for the motion of a viscous incompressible fluid in a two-dimensional non-uniform channel. Mekheimer (2004) studied the peristaltic flow of blood (obeying couple stress model) under the effect of magnetic field in non-uniform channels. He observed that the pressure rise for a couple stress fluids is greater than that for a Newtonian fluid. Also the pressure rise for uniform geometry is much smaller than that for non-uniform geometry. Hariharana et al. (2008) investigates the peristaltic transport of non-Newtonian fluid, modeled as power law and Bingham fluid, in a diverging tube with different wall wave forms. Mitra and Prasad (1973) analyzed the peristaltic motion of Newtonian fluid by considering the influence of the viscoelastic behaviour of walls. They assumed that the driving mechanism is in the form of a sinusoidal wave of moderate amplitude imposed on the flexible walls of the channel. Dynamic boundary conditions were proposed for the fluid motion due to the symmetric motion of the flexible walls which were assumed to be either thin elastic plates or membranes. Radhakrishnamacharya and Srinivasulu (2007) studied the influence of wall property on peristaltic transport with heat transfer. Sobh (2008) introduced slip effects on couple stress fluid. Ramana Kumari and Radhakrishnamacharya (2011) investigated the effect of slip on peristaltic transport in an inclined channel with wall effects. The influence of slip, wall properties on MHD peristaltic transport of a Newtonian fluid with heat transfer and porous medium have been investigated by Srinivas and Kothandapani (2009). Recently, the study of magneto-hydrodynamic (MHD) flow of electrically conducting fluids on peristaltic motion has become a subject of growing interest for researchers and clinicians. This is due to the fact that such studies are useful particularly for pumping of blood and magnetic resonance imaging (MRI). Theoretical work of Agarwal and Anwaruddin (1984) explored the effect of magnetic field on the flow of blood in atherosclerotic vessels of blood pump during cardiac operations. Li et al. (1994) observed that an impulsive magnetic field can be used for a therapeutic treatment of patients who have stone fragments in their urinary tract. Amit Medhavi (2008) studied Peristaltic Pumping of a Non-Newtonian Fluid. Amit Medhavi and U. K. Singh (2011) studied Peristaltic Induced Flow of a Particulate Suspension in a Non-Uniform Geometry

## 2. Formulation of the problem

We consider a peristaltic flow of a couple stress fluids in a symmetric channel with flexible walls, which are excited by a traveling longitudinal wave resulting in a peristaltic motion. An oscillatory time dependent flux is being imposed on then peristaltic flow. Choosing the Cartesian coordinate system  $O(x, y)$ , the flexible walls are represented by

$$y = \pm a_0 s \left[ \frac{X - ct}{\lambda} \right]$$

Where ‘ $a_0$ ’ is the wave amplitude, ‘ $c$ ’ is the wave velocity, ‘ $\lambda$ ’ is the wave length and ‘ $s$ ’ is an arbitrary function of the normalized axial coordinate.

$$x^* = \left[ \frac{X - ct}{\lambda} \right]$$

In the case of incompressible fluids when the body forces the body moments are absent, the equations of motion is

$$\rho \frac{DV}{Dt} = [-\nabla p + \mu \nabla^2 \mathbf{V} - \eta \nabla^4 \mathbf{V}]$$

The equations of motion for two-dimensional flow incompressible couple stress fluid in the component form are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.1}$$

$$\begin{aligned} & \rho \left[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right] \\ &= -\frac{\partial p}{\partial x} + \mu \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right] - \eta \left[ \frac{\partial^4 u}{\partial x^4} + \frac{\partial^4 u}{\partial y^4} + 2 \frac{\partial^4 u}{\partial x^2 \partial y^2} \right] - [\sigma B_0^2] u - \left[ \frac{\mu}{k_1} \right] u \end{aligned} \tag{2.2}$$

$$\begin{aligned} & \rho \left[ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right] \\ &= -\frac{\partial p}{\partial y} + \mu \left[ \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right] - \eta \left[ \frac{\partial^4 v}{\partial x^4} + \frac{\partial^4 v}{\partial y^4} + 2 \frac{\partial^4 v}{\partial x^2 \partial y^2} \right] - [\sigma B_0^2] v - \left[ \frac{\mu}{k_1} \right] v \end{aligned} \tag{2.3}$$

Where (u, v) are the velocity components along 0(x, y) directions respectively, p is the fluid pressure, ρ is the density of the fluid, μ is the coefficient of the viscosity, η is the coefficient of couple stress, σ is the electrical conductivity of the fluid, k<sub>1</sub> is the permeability of the porous medium and B<sub>0</sub> is the constant magnetic field. The imposed flux across the flexible channel is assumed to be Ψ<sub>f</sub>[1 + ke<sup>iωt</sup>], where Ψ<sub>f</sub> a characteristic flux, k is the amplitude of the flux and ω is the frequency of the oscillation.

The flow being two-dimensional in view of the incompressibility of the flow using (2.1) we introduce a stream function Ψ such that

$$\mathbf{u} = -\frac{\partial \Psi}{\partial y} \text{ and } \mathbf{v} = \frac{\partial \Psi}{\partial x} \tag{2.4}$$

Substituting (2.4) in (2.2) and (2.3) and eliminating p, the governing equations in terms of Ψ reduces to

$$\frac{\partial}{\partial t} [\nabla^2 \Psi] - [\Psi_y \nabla^2 \Psi_x] + [\Psi_x \nabla^2 \Psi_y] = \frac{\mu}{\rho} [\nabla^4 \Psi] - \frac{\eta}{\rho} [\nabla^6 \Psi] - \left[ \frac{\sigma B_0^2}{\rho} \right] \nabla^2 \Psi - \left[ \frac{\mu}{\rho k_1} \right] \nabla^2 \Psi \tag{2.5}$$

Where  $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$

The relevant conditions on Ψ are

$$\frac{\partial \Psi}{\partial x} = 0 ; \quad \frac{\partial^2 \Psi}{\partial y^2} = 0 \quad y = 0 \quad (2.6)$$

$$\Psi = \Psi_f [1 + k e^{i\omega t}] - a_0 c s \quad y = \pm a_0 s [x] \quad (2.7)$$

$$\frac{\partial^3 \Psi}{\partial y^3} = 0 \quad y = \pm a_0 s [x] \quad (2.8)$$

(2.6) guarantees the vanishing of the transverse flow on the axis of channel in view of the symmetry. (2.7) corresponds to the no slip of the axial velocity on the channel and also guarantees the assumption of the imposed oscillatory flux across the channel. (2.8) is the boundary condition related to couple stress fluids. We define the following non-dimensional variables

$$x^* = \left[ \frac{X - ct}{\lambda} \right] \quad y^* = \left[ \frac{Y}{a_0} \right] \quad t^* = [\omega t]$$

$$\Psi^* = \left[ \frac{\Psi}{a_0 c} \right] \quad \Psi_f^* = \left[ \frac{\Psi_f}{a_0 c} \right] \quad \varepsilon = \left[ \frac{a_0}{\lambda} \right]$$

Introducing these non- dimensional variables in (2.5) the governing equation in terms of  $\Psi^*$  reduces to (on dropping the asterisks)

$$-\text{Re} \varepsilon^3 \Psi_{xxxx} - \text{Re} \varepsilon \Psi_{yyy} - \text{Re} \varepsilon^3 \Psi_y \Psi_{xxx} - \text{Re} \varepsilon \Psi_y \Psi_{yyy} + \text{Re} \varepsilon^3 \Psi_x \Psi_{xyy} + \text{Re} \varepsilon \Psi_x \Psi_{yyy} = \left[ \varepsilon^2 \Psi_{xxxx} + \Psi \right] \quad (2.9)$$

$$R = \frac{\rho c a_0}{\mu} , \text{ Reynolds number}$$

$$S = \frac{\eta}{\rho c a_0^3} , \text{ Couple Stress Parameter}$$

$$M = \frac{\sigma B_0^2 a_0}{\rho c} , \text{ Magnetic Parameter}$$

$$D^{-1} = \frac{a_0^2}{k_1} , \text{ Inverse Darcy parameter}$$

### 3. Solution of the problem

Under long wave length assumption ( $\varepsilon \ll 1$ ) keeping in view of the condition (2.10),  $\Psi$  may be assumed in the form

$$\Psi = [\Psi_0 + k e^{it} \bar{\Psi}_0] + \varepsilon [\Psi_1 + k e^{it} \bar{\Psi}_1] \quad (3.1)$$

Substituting (3.1) in (2.9) and equating the like powers of  $\varepsilon$ , the equations corresponding to the zeroth order steady components are

$$RS \frac{\partial^6 \Psi_0}{\partial y^6} - \frac{\partial^4 \Psi_0}{\partial y^4} + [RM + D^{-1}] \frac{\partial^2 \Psi_0}{\partial y^2} = 0 \quad (3.2)$$

$$RS \frac{\partial^6 \bar{\Psi}_0}{\partial y^6} - \frac{\partial^4 \bar{\Psi}_0}{\partial y^4} + [RM + D^{-1}] \frac{\partial^2 \bar{\Psi}_0}{\partial y^2} = 0 \quad (3.3)$$

The conditions to be satisfied by  $\Psi_0$  and  $\bar{\Psi}_0$  are

$$\Psi_0 = 1 - S[x] \quad y = \pm S[x] \quad ; \quad \bar{\Psi}_0 = 1 \quad y = \pm S[x] \quad (3.4)$$

$$\frac{\partial^2 \Psi_0}{\partial y^2} = 0 \quad y = 0 \quad ; \quad \frac{\partial^2 \bar{\Psi}_0}{\partial y^2} = 0 \quad y = 0 \quad (3.5)$$

$$\frac{\partial \Psi_0}{\partial x} = 0 \quad y = 0 \quad ; \quad \frac{\partial \bar{\Psi}_0}{\partial x} = 0 \quad y = 0 \quad (3.6)$$

$$\frac{\partial^3 \Psi_0}{\partial y^3} = 0 \quad y = \pm S[x] \quad ; \quad \frac{\partial^3 \bar{\Psi}_0}{\partial y^3} = 0 \quad y = \pm S[x] \quad (3.7)$$

Solving (3.2) and (3.3) subject to the conditions (3.4-3.7), we obtain

$$\Psi_0 = C_1 + C_2 \text{Cosh}[\alpha_1 y] + C_3 \text{Cosh}[\alpha_2 y] \quad (3.8)$$

$$\bar{\Psi}_0 = C_4 + C_5 \text{Cosh}[\alpha_1 y] + C_6 \text{Cosh}[\alpha_2 y] \quad (3.9)$$

Where

$$C_1 = \left[ \frac{\alpha_2^2 [1 - S[x]]}{\alpha_1^2 [\text{Cosh}[\alpha_2 S[x] - 1]] - \alpha_2^2 [\text{Cosh}[\alpha_1 S[x] - 1]]} \right] - \left[ \frac{\alpha_1^2 [1 - S[x]]}{\alpha_1^2 [\text{Cosh}[\alpha_2 S[x] - 1]] - \alpha_2^2 [\text{Cosh}[\alpha_1 S[x] - 1]]} \right]$$

$$C_2 = \left[ \frac{\alpha_2^2 [S[x] - 1]}{\alpha_1^2 [\text{Cosh}[\alpha_2 S[x] - 1]] - \alpha_2^2 [\text{Cosh}[\alpha_1 S[x] - 1]]} \right]$$

$$C_3 = \left[ \frac{\alpha_1^2 [1 - S[x]]}{\alpha_1^2 [\text{Cosh}[\alpha_2 S[x] - 1]] - \alpha_2^2 [\text{Cosh}[\alpha_1 S[x] - 1]]} \right]$$

$$C_4 = \left[ \frac{\alpha_2^2}{\alpha_1^2 [\text{Cosh}[\alpha_2 S[x] - 1]] - \alpha_2^2 [\text{Cosh}[\alpha_1 S[x] - 1]]} \right] - \left[ \frac{\alpha_1^2}{\alpha_1^2 [\text{Cosh}[\alpha_2 S[x] - 1]] - \alpha_2^2 [\text{Cosh}[\alpha_1 S[x] - 1]]} \right]$$

$$C_5 = \left[ \frac{\alpha_2^2}{\alpha_2^2 [\text{Cosh}[\alpha_1 S[x] - 1]] - \alpha_1^2 [\text{Cosh}[\alpha_2 S[x] - 1]]} \right]$$

$$C_6 = \left[ \frac{\alpha_1^2}{\alpha_1^2 [\text{Cosh}[\alpha_2 S[x] - 1]] - \alpha_2^2 [\text{Cosh}[\alpha_1 S[x] - 1]]} \right]$$

$$\alpha_1 = \sqrt{1 + \frac{\sqrt{1 - 4RS(MR + D^{-1})}}{2RS}} \quad \alpha_2 = \sqrt{1 - 1 - \frac{4RS(MR + D^{-1})}{2RS}}$$

#### 4. Discussion of the results

In this paper, an attempt has been made to study analytically a mathematical model for the Magnetohydrodynamic peristaltic flow of a bio-fluid through a porous medium under the influence of low Reynolds number, considering the bio-fluid to be a couple stress fluids. Such a study possibly explains the pathological situations when a distribution of fatty cholesterol and artery clogging, blood clots are formed in the lumen of the coronary artery, which can be considered as equivalent to a fictitious porous medium. We have presented the graphical results of the solutions axial velocity  $u$ , Transverse velocity  $v$ . Figures (1) to (4) reveals axial velocity profiles for  $M = 0.1$  and  $S = 0.2$ ,  $R \geq 0.1$  and  $D^{-1} \geq 0.5$ . Figure (1) reveals axial velocity profile for  $M = 0.1$ ,  $S = 0.2$ ,  $R \geq 0.1$  and  $D^{-1} = 0.5$ , the resulting velocity in the converging (constricted) part of the channel is directed towards the boundary with the fluid moving in clockwise direction while in the dilated part, it moves in the anti-clockwise sense with resulting velocity directed towards the axis of the channel. Figure (2) corresponds to the velocity profile for  $M = 0.1$ ,  $S = 0.2$ ,  $R \geq 0.1$  and  $D^{-1} = 1$ . As already indicated that the flow separation takes place at all these values with fluid moving towards boundary in the clockwise sense in the constricted case and towards the axes in the anticlockwise sense in the dilated case (See Figure (1)), but in the dilated case the fluid flow drops out with higher speed. This is true even if  $M = 0.1$ ,  $S = 0.2$ ,  $R \geq 0.1$  and  $D^{-1} = 1.5$  (Figure 3), but in the constricted case the fluid flow enhances with higher speed, which is the contrast to the earlier case. Figures (4) to (6) reveals transverse velocity profiles for  $M = 0.1$  and  $S = 0.2$ ,  $R \geq 0.1$  and  $D^{-1} \geq 0.5$ , we observe that the flow separation takes place at all these values with fluid moving towards boundary in the clockwise sense in the constricted case and towards the axes in the anticlockwise sense in the dilated case (See figure (4)). But, in the dilated case the fluid flow drops out with higher speed and also the fluid flow enhances with higher speed in the constricted case (See figures (5) and (6)).

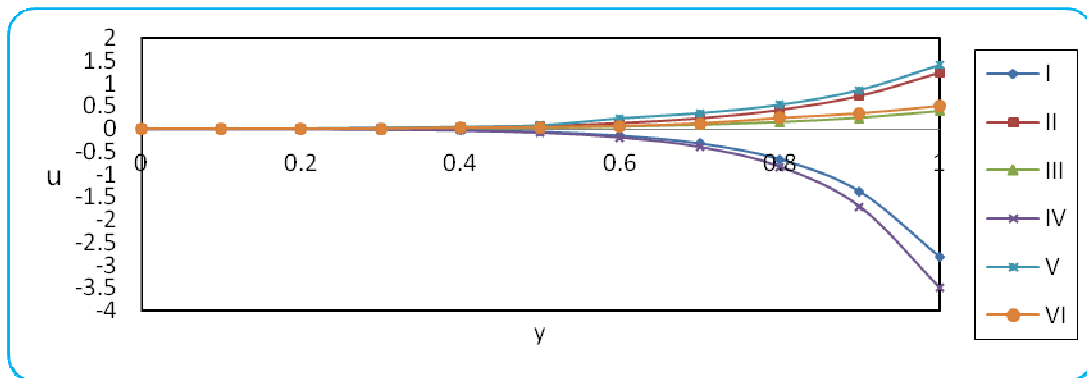
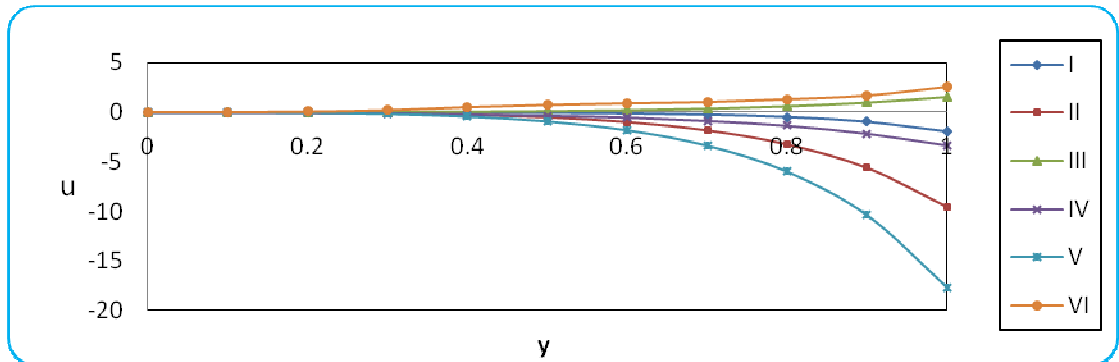


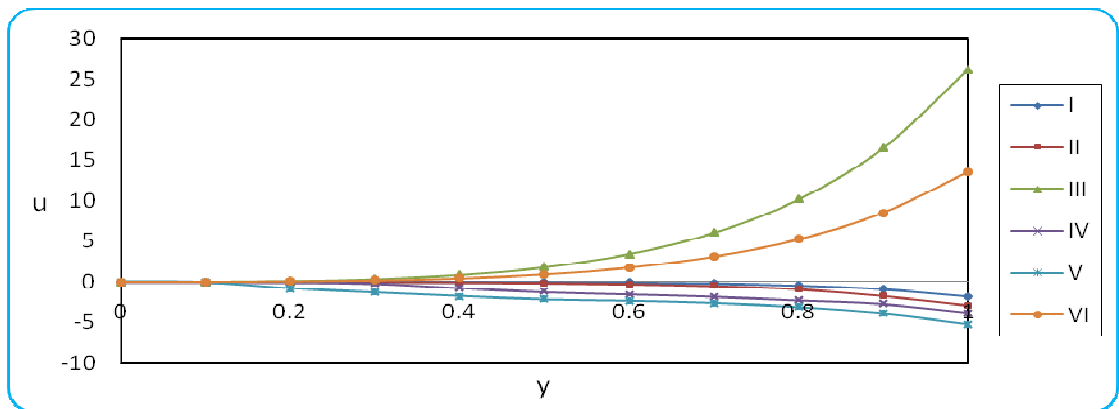
Figure (1):  $u$  with  $R$  when  $M=0.1, S=0.2, D^{-1}=0.5, C=0.01, k=0.1, x = t = \pi/6$

	I	II	III	IV	V	VI
R	0.1	0.2	0.3	-0.1	-0.2	-0.3
$\beta$	0.005	0.005	0.005	-0.005	-0.005	-0.005



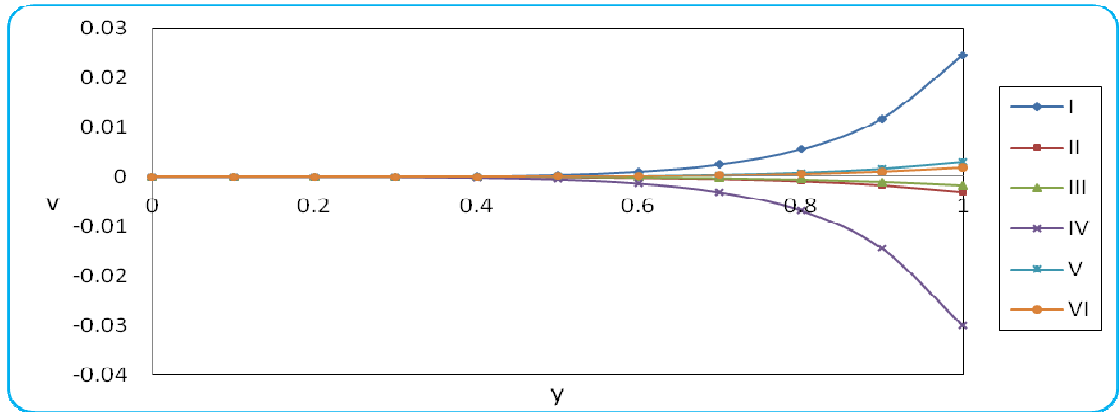
**Figure (2): u with R when  $M=0.1, S=0.2 D^{-1}=1 \epsilon=0.01, k=0.1, x = t = \pi/6$**

	I	II	III	IV	V	VI
R	0.1	0.2	0.3	-0.1	-0.2	-0.3
$\beta$	0.005	0.005	0.005	-0.005	-0.005	-0.005



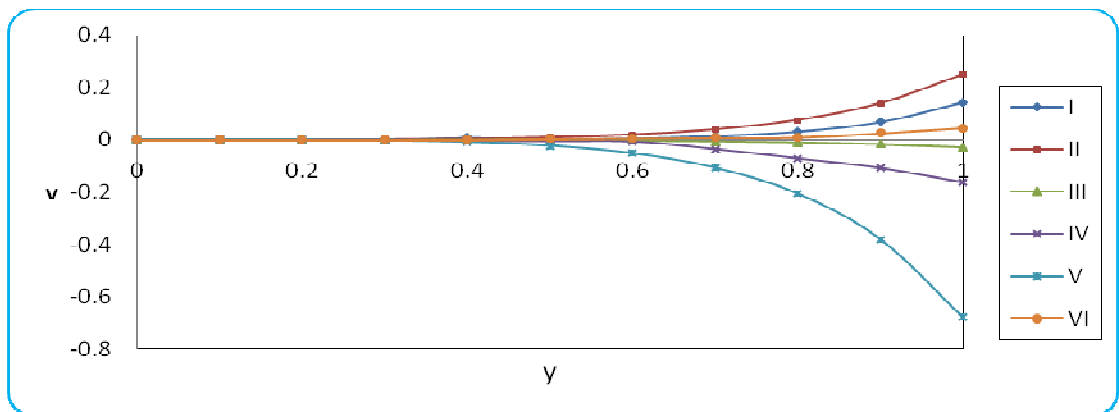
**Figure (3): u with R when  $M=0.1, S=0.2 D^{-1}=1.5, \epsilon=0.01, k=0.1, x = t = \pi/6$**

	I	II	III	IV	V	VI
R	0.1	0.2	0.3	-0.1	-0.2	-0.3
$\beta$	0.005	0.005	0.005	-0.005	-0.005	-0.005



**Figure (4): v with R when  $M=0.1, S=0.2 D^{-1}=0.5 C=0.01, k=0.1, x = t = \pi/6$**

	I	II	III	IV	V	VI
R	0.1	0.2	0.3	-0.1	-0.2	-0.3
$\beta$	0.005	0.005	0.005	-0.005	-0.005	-0.005



**Figure (5): v with R when  $M=0.1, S=0.2 D^{-1}=1, C=0.01, k=0.1, x = t = \pi/6$**

	I	II	III	IV	V	VI
R	0.1	0.2	0.3	-0.1	-0.2	-0.3
$\beta$	0.005	0.005	0.005	-0.005	-0.005	-0.005



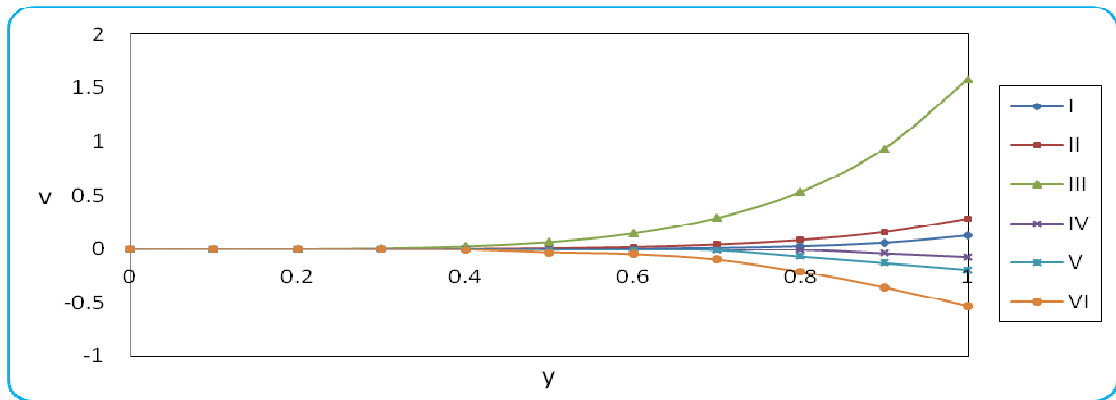


Figure (6): v with R when  $M=0.1, S=0.2, D^{-1}=1.5, C=0.01, k=0.1, x = t = \pi/6$

	I	II	III	IV	V	VI
R	0.1	0.2	0.3	-0.1	-0.2	-0.3
$\beta$	0.005	0.005	0.005	-0.005	-0.005	-0.005

### References

- Agarwal, H. L. and Anwaruddin, B. (1984). Peristaltic flow of blood in a branch, Ranchi Univ. Math. J, 15 , 111-121.
- AmitMedhavi (2008). Peristaltic Pumping of a Non-Newtonian Fluid, AAM: Intern, J., Vol. 3, No. 1, pp. 137-148.
- AmitMedhavi and Singh.U.K. (2011). Peristaltic Induced Flow of a Particulate Suspension in a Non-Uniform Geometry, AAM: Intern. J., Vol. 6, Issue 1 [Previously, Vol. 6, Issue 11, pp. 2064 – 2077], pp. 323 – 336
- Burns, J.C. and Parkes, T. (1969). Peristaltic motion, *J. Fluid Mech.*, **29**, pp. 731–743.
- Fung, Y.C. and Yih, C.S. (1968). Peristaltic transport, *Trans. ASME J. Appl. Mech.*, **35**, pp. 669–675.
- Jaffrin, M. Y. and Shapiro, A. H. (1971). Peristaltic Pumping, *Annual Rev. Fluid Mech.* 3, pp. 13-36.
- Latham, T. W. (1966). Fluid Motion in a Peristaltic Pump. M.S. Thesis, MIT, Cambridge.
- Li, A.; Nesterov, N. I.; Malikova, S. N. and Kiiatkin, V. A. (1994). The use of an impulsive magnetic field in the combined therapy of patients with stone fragments in the upper urinary tract, *Vopr. KurortolFizioter Lech FizKult*, 3 , 22-24.
- MaruthiPrasad, K. and Radhakrishnamacharya, G. (2007). Peristaltic transport of a Herschel-Bulkley fluid in a channel in the presence of magnetic field of low intensity, *Int. J. of Computational Intelligence and Applications*, **1**, pp. 71–81, 2007.
- Mekheimer, Kh S. (2004). Peristaltic flow of blood under the effect of magnetic field in a non-uniform channel, *Appl. Math. Comput.* Vol.153, pp 763–77.
- Misra, M. and Rao, A.R. (2003). Peristaltic transport of a Newtonian fluid in an asymmetric channel, *Z. Angew. Math. Phys.*, **54**, pp. 532–550.
- Mitra, T. K. and Prasad, S. N. (1973). On the influence of wall properties and Poiseuille flow in the peristalsis, *J. Biomechanics* vol.6, pp. 681–693.
- Muthu, P.; RathishKumar, B.V. and Peeyush Chandra. (2007). Peristaltic motion of micropolar fluid in circular cylindrical tubes: Effect of wall properties, *Appl. Math. Model.*, accepted, 2007.

- PrasannaHariharan; Seshadri, V.; Rupak, K and Banerjee.(2008). Peristaltic transport of non-Newtonian fluid in a diverging tube with different wave forms, *Mathematical and Computer Modelling* vol. 48, pp. 998–1017
- RamanaKumari, A.V. and Radhakrishnamacharya, G. (2011). Effect of slip on peristaltic transport in an inclined channel with wall effects, *Int. J. of Appl. Math and Mech.* Vol. **7** (1), pp. 1- 14.
- Radhakrishnamacharya, G. and Radhakrishna Murthy, V.(1993). Heat transfer to peristaltic transport in a non-uniform channel, *Defence Science Journal.* Vol. 43(3), pp. 275-280.
- Radhakrishnamacharya,G. and Srinivasulu,Ch.(2007). Influence of wall properties on peristaltic transport with heat transfer, *C. R. Mechanics*, **335**, pp. 369–373.
- Selverov,K.P.and Stone,H.A.(2001).Peristaltically driven channel flows with applications toward micromixing, *Phys. Fluids*, **13**, pp. 1837–1859.
- Shapiro,A.H.; Jaffrin,M.Y. and Weinberg,S.L. (1969). Peristaltic pumping with long Wave lengths at low Reynolds numbers, *J. Fluid Mech.*, **37**, pp. 799–825.
- Sobh, A.M.(2008). Interaction of couple stresses and slips flow on peristaltic transport in uniform and non-uniform channels, *Turkish J. Eng. Environ.Sci.* vol. 32, pp.117-123.
- Srinivas. S.; Gayathri, R. and Kothandapani, M. (2009).The influence of slip conditions, wall properties and heat transfer on MHD peristaltic transport. *Computer Physics Communication*, vol.180, pp. 2115–22.
- Srivastava, L.M. and Srivastava, V.P. (1984). Peristaltic Transport of Blood: Casson ModelIII. *Journal of Biomechanics*, Vol. 17, pp. 821-829.
- Srivastava, L. M. and Srivastava, V. P.(1988). Peristaltic Transport of a Power-Law Fluid: Application to the DuctusEfferentes of the Reproductive Tract, *Rheol. Acta*, vol.27, pp. 428–433.
- Srivastava, L. M.; Srivastava, V. P. and Sinha, S. N. (1983).Peristaltic Transport of a Physiological Fluid: Part I. Flow in Non–Uniform Geometry, *Biorheol.* Vol. 20 pp.153–166.
- Xiao, Q. and Damodaran,M.(2002).A numerical investigation of peristaltic waves in circular tubes, *Int. J. Comput. Fluid Dyn.* **16**, pp. 201–216.